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Mathematical Games and Sampling Inspection Plans

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Solutions to quality control by lot sampling through the game theory approach are presented, and the results are compared with those obtained by the classical statistical method. Single and double plans are considered and modeled as two-person zero-sum games, and optimal solutions are found. Most of the solutions are reminiscent of known statistical results and reinforce them by adding new features.

KEY WORDS: Two-person zero-sum mathematical games; quality control; quality batch; single-sampling inspection plans; double-sampling inspection plans.

1. INTRODUCTION

Game theory has been shown to be a useful tool mainly for solving problems in economic and military behavior. Recently, the Nobel Prize in economics was conferred on John Nash, John Harsanyi, and Reinhard Selten for their achievements in applying game theory to economics.

In most practical problems a complete solution is usually provided by several theories or approaches. The classical approach for solving problems of quality control is the statistical one. Here, we present solutions to quality control by lot sampling, through the game theory approach.

Quality control is one of the major problems in technology. *Quality* is a conformance to specifications, and the degree of conformance is the measure of quality. Quality control procedures are divided into (i) control of *variables* and (ii) control of *attributes*.

A *variable* is an item product characteristic measurable on a continuous scale, and therefore may have fractional values. An *attribute* is a

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characteristic which is either possessed or not by an item, and it is always an integer number. The diameter of a shaft and the carbon content of steel are examples of variables, while acceptable or rejected products, undersize, on size, or oversize products are examples of attributes. Attribute control is generally an after-production fact control, i.e., the manufactured product is classified as being either acceptable or unacceptable. A control variable may be an after or before the production fact, as it is occasionally possible to measure and change the item characteristic while the process is running.

When the quality characteristic has to be counted rather than measured, attributes are involved rather than variables.

The term *defect* or *defective* is used when dealing with attributes, and denotes faults or errors of an item which cause it to be unacceptable. An unacceptable item may have one or more defects. Let us consider, for example, a hole, with a prescribed specification, being inspected by a plug gauge with one end able and the other one unable to enter the hole. With such a gauge one may sort the product into: on size, oversize, or undersize, or simply acceptable or unacceptable. In either case we do not record the actual dimensions, but count the number of rejected items in each sample and express them as a fraction (or as a percentage) of the inspected number. This is perhaps the most common approach and permits the use of a binomial distribution in analyzing the results.

The binomial probability of any specific number of defectives in a sample of any size is

$$P(r \mid n, p) = \frac{n!}{r!\,(n-r)!}\, p^r q^{n-r} \tag{1}$$

and the probability of the occurrence of a defective is

$$\bar{p} = \frac{\sum r}{\sum n} \qquad \text{or} \qquad \bar{p} = \frac{\sum P}{\text{number of samples}} \tag{2}$$

where r is the number of times an event occurs, i.e., the number of rejected items for a single sample, n is the number of trials, i.e., number of inspected items ($n > r$), p is the probability that the event will occur, i.e., a defective item will be found, and $q = 1 - p$ is the probability that the event will not occur, i.e., the item will be acceptable.

The probability of obtaining at most r defectives is a cumulative probability, equal to the sum $P(0) + P(1) + P(2) + \cdots + P(r)$ (see, for example, ref. 2). Formally, if X is a discrete random variable, with each possible outcome r in the range $\{0, 1, 2, \ldots\}$, then the probability that X assumes a value less than or equal to a number x is

$$F(x) = P(X \leqslant x) = \sum_{r=0}^{x} P(r) \tag{3}$$

where $F(x)$ is the cumulative distribution function of the random variable X.

The most common inspection procedure is to arrange the products in batches or lots and select at random a sample from each batch. A decision of accepting or rejecting the batch as a whole is made on the basis of the sampling analysis result. If the judgment is made from a sample result, there is always a risk of accepting a bad batch and rejecting a good one. The efficiency of the judgment depends primarily on the number of items inspected; the larger the sample, the better the precision of sorting into good and bad batches, and the smaller the proportion of necessary sampling.

Sampling plans may be divided into the following types: (i) *single*, (ii) *double*, and (iii) *multiple*. A *single-sampling plan* is defined by the size of the lot (N), the size of the sample (n), and the acceptance number (c). For example, the plan defined by $N = 900$, $n = 90$, $c = 2$ means that 90 units were inspected from a total of 900, and if two or less defectives are found the entire lot is accepted; if three or more defectives are found in the 90-unit sample, the entire lot is rejected.

Double-sampling plans are somewhat more complicated. On the initial sample, a decision based on the inspection results is made whether (i) to accept the lot, (ii) to reject the lot, or (iii) to take another sample. If a second sample is required, the results of that inspection and of the first inspection are used to reject or accept the lot. A double-sampling plan is defined by the size of the lot (N), the size of the sample in the first sample (n_1), the acceptance number in the first sample (c_1), the rejection number in the first sample (r_1), the sample size on the second sample (n_2), the acceptance number for both samples (c_2), and the rejection number for both samples (r_2).

If the values of r_1 and r_2 are not prescribed, they are equal to $c_2 + 1$. An illustrative example is as follows: let $N = 900$, $n_1 = 90$, $c_1 = 1$, $r_1 = 5$, $n_2 = 200$, $c_2 = 6$, and $r_2 = 7$. An initial sample size equal to 90 is selected from the lot of 900 and inspected. One of the following judgments is made: (i) If there are 1 or less defectives (c_1), the lot is accepted, (ii) if there are 5 or more defectives (r_1) the lot is rejected, and (iii) if there are 2, 3, or 4 defectives, no decision is made, and a second sample is considered.

A second sample of 200 (n_2) from the lot (N) is inspected, and one of the following judgments is made: (i) If there are 6 or less defectives (c_2) in both samples, the lot is accepted. This number is obtained by 2 in the first sample and 4 or less in the second sample, by 3 in the first sample and 3 or less in the second sample, or by 4 in the first sample and 2 or less in the second sample. (ii) If there are 7 or more defectives (r_2) in both samples, the lot is rejected. This number is obtained by 2 in the first sample and 5 or more in the second sample, by 3 in the first sample and 4 or more in the second sample, or by 4 in the first sample and 3 or more in the second sample.

The efficiency of any sampling scheme as a detector of good and bad batches can be represented by means of its *operating characteristic* (OC), in which the vertical scale presents the chance of accepting a batch of the quality specified on the horizontal scale.

2. QUALITY CONTROL BY SAMPLING PLANES AS A TWO-PERSON ZERO-SUM GAME

The traditional method in treating problems of quality control is the statistical one. It is interesting to try solving the same problems by other approaches, to compare the results, and possibly to find new aspects than those achieved through statistical methods.

Game theory is concerned with those situations in which the result of the choice made by a person depends on the choice made by another person. A number is attributed to each result and the established gain (reward or benefit) stands for the value granted by the player to the result of his or her choice.

The aim of an ideal player in game theory is to obtain the greatest profit, having in view that the other players have similar aims.

A game is defined if: (i) There are at least two players. (ii) The game starts when one or more players makes a choice from many precise possible choices. (iii) The first choice, as well as any other choice, leads to a certain situation which, in turn, induces the player who must make the next choice as well as all choices corresponding to this player (in this situation). In any case, for each game, it is surely known who must play and what the choices are during the whole development of the game. (iv) The choices of a player may be known or not by the other players. A game in which each player's choice is immediately known by all players is called a *game with complete information*. Chess is an example of a game with complete information, while almost all card games are not. This notion is important because there is always an optimal method for solving such a game, without resort to chance. (v) If a game is described by its consecutive situations, then there is a rule which specifies the end of the game. (vi) Each game ends in a situation which confers a reward or loss for each engaged player.

Any particular instance of a game is called a *play* (elementary game). The rules of the game, which are known to the players, specify the set of pure strategies available to each of the players.

A *pure strategy* is a plan formulated by a player prior to a play, which will cover all the possible decisions which may be confronted during any play of the game. A *mixed strategy* is a probabilistic choice from the set of pure strategies. The *optimal strategy* yields the best of the possible outcomes. An optimal solution is said to be reached if no player finds it beneficial to alter strategy. In this case, the game is said to be in a *state of equilibrium*.

A game with two players in which any gain of one player equals a corresponding loss of the other player is known as a *two-person zero-sum game*. In such a game, it suffices to express the outcomes (the rewards or the payoff function values) in terms of the payoff to one player. The optimal solution of a two-person zero-sum game is given by the *minimax criterion* (to be explained below) which selects for each player a strategy· yielding the best of the worst possible outcomes. Thus the minimax criterion accommodates the fact that each player is acting against the other's interest.

In the present paper, we have in view those games in which the number of pure strategies is finite. Then, the game can be described by an M by N matrix $C=[c_{ij}]$, where each entry c_{ij} represents the amount that the first player (in the present case the searcher or the inspector) receives from the second (in this work the inspected lot, which symbolizes the producer) if the first player uses his ith pure strategy and the second player uses his jth pure strategy. It may happen that the matrix C will have a *saddle point*, i.e., an element $c_{i'j'}$ such that

$$\max_{1 \leqslant i \leqslant m} c_{ij'} = c_{i'j'} = \min_{1 \leqslant j \leqslant n} c_{i'j} \tag{4}$$

In this case, the game would be in a state of equilibrium if the first player chooses his i'th pure strategy and the second player chooses his j'th pure strategy. However, usually such a saddle point does not exist and we are forced to use mixed strategies, denoted by $X=(x_1, x_2, ..., x_m)$ and $Y=(y_1, y_2, ..., y_n)$, respectively, where x_i, $i=1, ..., m$, is the probability that the first player will choose his ith pure strategy and y_j, $j=1, ..., n$, is the probability that the second player will choose his jth pure strategy.

When the first player plays a mixed strategy X and the second player a mixed strategy Y, the expected payment is given by the function

$$c(X, Y) = \sum_i \sum_j c_{ij} x_i y_j \tag{5}$$

The fundamental theorem of two-person zero-sum finite games states that

$$\max_X \min_Y c(X, Y) = \min_Y \max_X c(X, Y) \tag{6}$$

This minimax value of c is called the *value* (v) of the game.

Searching for an optimal sampling plan to decide the acceptance of a given lot is similar to a decision problem in a two-person zero-sum game. The inspector and the incoming batch may be considered the two players of the game. There is a finite number of moves or pure strategies in the hands of each player; the former may use a finite number of sampling plans, while the latter may be *effective* or *defective*, or may have different incoming qualities. A solution for such a game would help the inspector to decide what kind of sampling inspection has to be performed to obtain an optimal response concerning the acceptance of a certain batch. The probability of a batch (or lot) acceptance may serve as a *payoff function* and the binomial probability distribution may be used to calculate its values. An optimal solution for a two-person zero-sum game is given by the *minimax criterion*, [6,5] which selects for each player a strategy yielding the best of the worst possible outcomes.

The method is exemplified below for particular data sets, trying to answer a number of problems concerning single- or double-sampling plans, and, when possible, to compare the results with those obtained by statistical methods.

2.1. Single-Sampling Plans

Example 1. Let us suppose an inspector having to inspect a batch of a good quality ($p_1 = 0.01$ or $p_2 = 0.02$) containing 600 units. He chooses, for example,[7] three sampling plans, the first with $n_1 = 50$ and $c_1 = 0$, the second with $n_2 = 50$ and $c_2 = 1$, and the third with $n_3 = 100$ and $c_3 = 3$. In other words, the two players of this *game* are the inspector with his three strategies corresponding to the three sampling plans previously mentioned, and the inspected batch with its two strategies, i.e., the proportion qualities $p_1 = 0.01$ and $p_2 = 0.02$. Table I presents the payoff matrix of this game, a 3×2 matrix with entries calculated by means of the Poisson table and representing the probabilities of the batch acceptance. The Poisson distribution is a limiting form of the binomial distribution and may be successfully used instead of the latter when n is large in comparison to N and p approaches zero. We have in view the following probability;

$$P(x) = \frac{\lambda x^{\lambda}}{x!}, \qquad x = 0, 1, 2, \ldots \tag{7}$$

Table I. The Payoff Matrix of the Game in Example 1[a]

	p_1	p_2
s_1	0.61	0.37
s_2	0.91	0.74
s_3	0.98	0.86

[a] Each (i, j) entry, $i \in \{1, 2, 3\}$, $j \in \{1, 2\}$, represents the acceptance probability of a p_j-quality batch when the s_i sampling plan is used.

where x represents the number of occurrences of the same event in n independent trials and $\lambda = np$ represents the mean.

One may observe that the first pure strategy s_1 is dominated by the other two strategies, s_2 and s_3, that is, the gain corresponding to the pair (s_1, p_1) is smaller than the gains corresponding to the pairs (s_2, p_1) or (s_3, p_1), and the gain corresponding to the pair (s_1, p_2) is smaller than the gains corresponding to the pairs (s_2, p_2) or (s_3, p_2), respectively. It follows that we can eliminate the first strategy s_1 as being *a priori* disadvantageous, and reduce the game matrix to a 2×2 square matrix, as shown in Table II. This new matrix has a *saddle point* 0.86, that is, an element which is the minimum in its row and the maximum in its column. As is well known (for example, ref. 5), such a special point is called *the value of the game* and the corresponding pair of pure strategies is called the *optimal solution* of the game. We have in view the pair of strategies (s_3, p_2) and its corresponding payoff or gain (matrix entry), which equals 0.86. In other words, the optimal solution of our imaginary game is obtained when the inspector chooses his third strategy, while the received batch has 2% defectives. Rosander's [9] statistical results are: (i) When the lot size N and the sample size n remain unchanged, the first sampling plan, whose acceptance number

Table II. The Reduced Payoff Matrix of the Game in Example 1[a]

	p_1	p_2
s_2	0.91	0.74
s_3	0.98	0.86

[a] This matrix was obtained by eliminating the first row in the matrix in Table I.

is 0, is the worst. Such a plan rejects too high a proportion of acceptable quality lots, contrary to intuition and common sense. (ii) If N remains fixed while the sample size as well as the acceptance number are increased, we obtain the best sample plan of the three, i.e., s_3.

Let us recall our previous results. We rejected the first strategy s_1 (the first sampling plan) as being *a priori* disadvantageous from the game theory viewpoint, and selected the third strategy s_3 (the third sampling plan) as an optimal strategy in detecting a lot of a good quality. In other words, our results reinforce the statistical conclusions by adding new motivation to choose the third single-sampling plan as the best inspection plan in the case of a good-quality lot.

Example 2. It is interesting to see what happens when the same problem is raised for a 600-unit batch of relatively poor quality, that is, $p_1 = 0.07$ and $p_2 = 0.08$. After Rosander,[7] the OC curves associated with the three sample plans show that only small proportions of such batches are accepted by the first plan, where the acceptance number equals zero, the second plan, with the acceptance number 1, is not an improvement on the first, while the third plan, with the acceptance number 3, has an acceptance level between that of the other two plans.

From the point of view of game theory, we have a new game where the strategies of the inspector remain unchanged, while the batch has at its disposal two new strategies, one for the proportion defective $p_1 = 0.07$ and the other for $p_2 = 0.08$. Table III represents the normal form of the game. As in Table I, we can eliminate the first strategy s_1, since it is dominated by the third strategy s_3. Thus, we obtain a simplified normal form of the game, represented by the 2×2 matrix in Table IV. We may conclude that the pair of strategies (s_2, p_2) is a saddle point of our second game. In other words, an optimal solution of such a game is obtained when for a 600-unit lot with 0.08 defectives, the inspector will use his second sample plan, that

Table III. The Payoff Matrix of the Game in Example 2[a]

	p_1	p_2
s_1	0.03	0.04
s_2	0.14	0.09
s_3	0.08	0.04

[a] Each (i, j) entry, $i \in \{1, 2, 3\}$, $j \in \{1, 2\}$, represents the acceptance probability of a p_j-quality batch when the s_i sampling plan is used.

Table IV. The Reduced Payoff Matrix of the Game in Example 2[a]

	p_1	p_2
s_2	0.14	0.09
s_3	0.08	0.04

[a] This matrix was obtained by eliminating the first row in the matrix in Table III.

is, $n_2 = 50$ and $c_2 = 1$. One should bear in mind that an optimal solution shows the best of the worst outcomes of a game, taking into account that the two players have opposite interests and each player is acting against the other's interest. Returning to the statistical results in Example 2, one may observe the similarity between these results and the present ones based on a game theory approach. The same second sampling plan, s_2, appears to be the most advantageous in looking at poor-quality batches.

Example 3. We can imagine a new game with the same players as in the previous two games and the same strategies for the inspector, but with different strategies for the batch. We have in view a new situation when we do not know the batch (lot) quality, but we know the operating characteristic curves associated with the three sampling plans. Since such curves are asymptotic to the percent defective axis, we can determine the limiting value for the incoming perçent defective (batch quality) and thus establish the batch "strategies."

If we continue to discuss Rosander's[7] example of single-sampling plans for the 600-unit batch, we can presume that the incoming lot has at its disposal four strategies, namely $p_1 = 0.01$, $p_2 = 0.03$, $p_3 = 0.05$, $p_4 = 0.07$.

Table V. The Payoff Matrix of the Game in Example 3[a]

	p_1	p_2	p_3	p_4
s_1	0.61	0.22	0.08	0.03
s_2	0.91	0.56	0.29	0.14
s_3	0.98	0.65	0.27	0.08

[a] Each (i, j) entry, $i \in \{1, 2, 3\}$, $j \in \{1, 2, 3, 4\}$, represents the acceptance probability of a p_j-quality batch when the s_i sampling plan is used.

Table VI. The Reduced Payoff Matrix of the Game in Example 3[a]

	p_3	p_4
s_2	0.29	0.67
s_3	0.27	0.08

[a] This matrix was obtained by eliminating the first row and the first and second columns in the matrix in Table V.

In other words, the received lot may be of good, intermediate, or poor quality. The normal form of this third game is represented by the 3×4 matrix (Table V). As in the previous two games, the first strategy s_1 of the inspector must be eliminated as being *a priori* disadvantageous, since it is dominated by the strategies s_2 and s_3. The remaining 2×4 matrix may be reduced to a 2×2 matrix (Table VI) by eliminating the first two strategies of the incoming lot as being *a priori* disadvantageous. Indeed, p_1 and p_2 are dominated by p_3 and p_4. This last matrix has a saddle point, representing an optimal solution of the third imaginary game, namely the pair of strategies (s_2, p_4) and the game value equals 0.14. Thus, the incoming batch will select the poorest quality from its quality range (the set of strategies p_1, p_2, p_3, p_4). The inspector does not know the true quality of the receiving lot, but he knows its range and if he wants to give a greater chance to an intermediate-quality batch to be accepted, he has to select his second sampling plan. According to our best knowledge, there is no statistical example similar to the present considered case. Thus, we could not compare our result with any statistical one.

In the present section we considered three different mathematical games to model three different situations of an incoming lot of items; in all cases the inspector has at his disposal three sampling plans. These games are two-person zero-sum games, and it is not clear if they are of complete or incomplete information. Since all their associated matrices may be reduced to a single element by eliminating the dominated strategies,[5] it follows that all previous games are with complete information and that in such cases there is always an optimal solution.

2.2. Double-Sampling Plans

Example 4. Let us model a double-sampling plan as a two-person zero-sum game. The considered example is presented in Besterfield[3] with the following data: $N = 2400$, $n_1 = 150$, $c_1 = 1$, $r_1 = 4$, $n_2 = 200$, $c_2 = 4$, $r_2 = 5$.

Table VII. The Payoff Matrix of the Game in Example 4

	b_1	b_2
a_1	0.558	0.442
a_2	0.221	0.779

In our imaginary game the inspector is one of the players and the lot with 2400 units and 0.01 fraction defective is the other one. Suppose that this inspector selected the previous double–sample plan, such that there are two pure strategies at his disposal: a_1 representing the single-sample plan with the data n_1, c_1, and r_1; and a_2, representing the single-sample plan with the data $n_1 + n_2$, c_2, and r_2. The inspected lot, as an immobile player, has at its disposals two pure strategies: b_1 to be accepted and b_2 to be rejected. The normal form of the game is represented by the matrix in Table VII, such that each position (i, j) contains the acceptance probability of the incoming batch (this time, with 1% defectives only). The acceptance probability is calculated by means of the Poisson probability, taking into consideration the lot quality (proportion defective) and the sample size in each strategy. For example, the payoff associated with the pair of strategies (a_1, b_1) represents the acceptance probability of a 2400-unit lot with 0.01 proportion defectives when a 150-unit sample is randomly chosen from the lot with the acceptance number $c_1 = 1$. The payoff corresponding to the pair of strategies (a_2, b_1) represents the acceptance probability of the given lot when the combined samples (150 units + 200 units) are used with the acceptance number $c_2 = 4$. Symbolically, these two probabilities are obtained from the following two equations:

$$(P_a)_1 = (P_{1 \text{ or less}})_1 \tag{8}$$

$$(P_a)_2 = (P_2)_1 \, (P_{2 \text{ or less}})_2 + (P_3)_1 \, (P_{1 \text{ or less}})_2 \tag{9}$$

The matrix in Table VII has no saddle point, that is, the pure maximin (maximum of rows minimum) is different from the pure minimax (minimum of columns maximum). In other words, the game value v stands between 0.442 and 0.558. This difficulty may be avoided by choosing a pure strategy regulated by chance, i.e., a *mixed strategy*. To obtain the optimal strategy of the inspector we must solve the following system of equations:

$$\begin{aligned} a_{11} p_1 + a_{21} p_2 &= v \\ a_{12} p_1 + a_{22} p_2 &= v \end{aligned} \tag{10}$$

Each a_{ij} represents a 2×2 matrix entry, v stands for the value of the game, and $p_1 + p_2 = 1$, where p_1 and p_2 are probabilities.

In the considered particular case one must solve the system

$$\begin{aligned} 0.558p_1 + 0.221p_2 &= v \\ 0.442p_1 + 0.779p_2 &= v \end{aligned} \tag{11}$$

and obtain $p_1 = 0.827$, $p_2 = 0.173$, and $v = 0.5$. In other words, an optimal mixed strategy of the inspector is obtained when he uses its first pure strategy with the probability 0.827 and its second pure strategy with the probability 0.173. This result recalls that the first sample in a double-sample plan is preferentially used in a lot-by-lot sampling inspection.

To obtain an optimal strategy for the incoming lot, one must solve the second system of equations

$$\begin{aligned} 0.558q_1 + 0.442q_2 &= v \\ 0.221q_1 + 0.779q_2 &= v \end{aligned} \tag{12}$$

As previously mentioned, v represents the value of the game and q_1 and q_2 are probabilities such that $q_1 + q_2 = 1$. One obtains $q_1 = q_2 = 0.5$. In other words, an optimal strategy of the received lot is that in which the batch is accepted or rejected with the same probability 0.5. An optimal mixed strategy for the inspector and an optimal mixed strategy for the given lot form a pair of strategies representing an optimal solution of the proposed game.[3,6]

3. CONCLUSIONS

Game theory may contribute to studies of conflict and solve problems of practical interest (e.g., refs. 2 and 4). Some aspects of the theory were developed in the present investigation and applied to problems of quality control by lot sampling.

The obtained results have added new aspects to the classical statistical results of quality control by lot sampling. In the statistical approach, the type of plan for a particular producer or product is based not only on its effectiveness, but also on additional aspects such as administrative costs, quality information, number of units inspected, and psychological impact. The game theory approach takes into account mainly mathematical criteria and results; it does not use visual tools such as diagrams. As proved in the present investigation, using the game theory approach one reaches similar results to the classical statistical one, proving that there is always an optimal inspection plan to detect a lot of an incoming quality (percentage of defectives).

It has been shown in the literature that usually there is more than one possibility of interpreting the results of a game-theory analysis, and that some of the interpretations are more useful than others. In particular, there is some merit in achieving saddlepoint solutions whenever possible. The present paper considers only a very small part of the quality control problems. It should be interesting to study, for example, problems of multiple-sampling plans and problems of control plans for variables. It also should be interesting and surely most efficient to write and implement computer programs to verify the existence of and find (when it is the case) the saddlepoint of a normal matrix game associated with a quality control problem.

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